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## VACUUM FLUCTUATIONS AND DECOHERENCE IN MESOSCOPIC AND MICROSCOPIC SYSTEMS

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We discuss recent experimental evidence of decoherence in a laboratory mesoscopic system in a cavity, from which we draw analogies with the decoherence that we argue is induced by microscopic quantum-gravity fluctuations in the space-time background. We emphasize the parallel rôles played in both cases by dissipation through non-trivial vacuum fluctuations that trigger the collapse of an initially coherent quantum state. We review a phenomenological parametrization of possible effects of this kind in the neutral kaon system, where they would induce CPT violation, and describe some experimental tests.

### 1 Introduction and Summary

There is currently much debate whether microscopic black holes induce quantum decoherence at a microscopic level. In particular, it has been suggested <sup>1</sup> that Planck-scale black holes and other topological fluctuations in the space-time background cause a breakdown of the conventional  $S$ -matrix description

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of asymptotic particle scattering in local quantum field theory, which should be replaced by a non-factorizable superscattering operator  $\mathcal{S}$  relating initial- and final-state density matrices:

$$\rho_{out} = \mathcal{S} \rho_{in} \quad (1)$$

It has further been pointed out that, if this suggestion is correct, there must be a modification of the usual quantum-mechanical time evolution of the wave function, taking the form of a modified Liouville equation for the density matrix<sup>2</sup>:

$$\partial_t \rho = \frac{i}{\hbar} [\rho, H] + \mathcal{J} H \rho \quad (2)$$

The extra term in (2) is of the form generally encountered in the description of an open quantum-mechanical system<sup>3,4</sup>, in which observable degrees of freedom are coupled to unobservable components which are effectively integrated over, which may evolve from a pure state to a mixed state with a corresponding increase in entropy. Any such evolution entails a violation of CPT, though in a different form from that sometimes proposed in the context of conventional space-time quantum mechanics<sup>5 b</sup>.

The necessity of a mixed-state description is generally accepted in the presence of a macroscopic black hole, but is far from being universally accepted in the case of microscopic virtual black-hole fluctuations. In a research programme to elucidate this question, we have been analyzing the possibility of quantum decoherence in a non-critical formulation of string theory<sup>7</sup>, and indeed found an extra term in the quantum Liouville equation of the form conjectured in (2). In the absence of a satisfactory treatment of quantum gravity, the results of our work can only be regarded as indicative. However, we think that they constitute interesting circumstantial evidence in favour of the picture advanced previously<sup>1,2,7</sup>, namely that microscopic quantum fluctuations in the space-time background may induce a loss of quantum coherence in apparently isolated systems. Moreover, the magnitude of  $\mathcal{J}H \sim \mathcal{O}(\frac{E^2}{M_P^2})$ , with  $E$  a typical low-energy scale, that we find is consistent with previous string estimates, and may not lie many orders of magnitude beyond the reach of particle physics experiments in the neutral kaon system<sup>8,9,10</sup>, that are sensitive to this form of decoherence and the related CPT violation<sup>c</sup>.

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<sup>b</sup>Proposals for violations of CPT within the quantum-mechanical context of pure states have also been made recently in the context of critical string theory<sup>6</sup>. In particular, the authors of ref.<sup>6</sup> have proposed certain perturbative backgrounds of critical strings that violate Lorentz invariance and hence CPT.

<sup>c</sup>We stress that the approach advocated in ref.<sup>2,7</sup> is different from the model for quantum measurement proposed in ref.<sup>11</sup>, which involves a reduction of the wavefunction by occa-

In this talk we review briefly the formalism we propose, as well as the present experimental limits on this form of CPT violation. Before doing so, however, we first review recent experimental results<sup>12</sup> confirming the rôle of the environment in inducing decoherence in mesoscopic systems, which confirm theoretical expectations for the rôle of the environment in the transition from the quantum to classical worlds<sup>4</sup>, and bear close analogy with the quantum-gravity phenomena that we advocate.

## 2 Schrödinger's Cat in the Laboratory

We start by reviewing the experimental situation concerning the preparation of 'Schrödinger's cats' in the laboratory, basing our discussion on<sup>12</sup>. To understand the physics behind the construction, we find it instructive to define exactly what these 'beasts' are, and how they can be constructed experimentally. There is a huge literature in Quantum Optics on this, which originated from special studies of the behaviour of atoms in electromagnetic cavities<sup>13</sup>. Schrödinger's cat is prepared by letting an atom, which may be in a quantum superposition of two states  $e, g$ , pass through a cavity containing (quantized) electromagnetic radiation. The coupling of the atom to coherent cavity modes of radiation has been tested experimentally in recent years<sup>14</sup>. This coupling manifests itself through the so-called 'vacuum Rabi splitting', i.e., a splitting of the spontaneous emission<sup>15</sup> or absorption<sup>16</sup> spectra of atoms inside cavities containing coherent electromagnetic radiation, as a result of the interaction of the atom with coherent cavity modes.

It is instructive to review a quantum-mechanical derivation of the Rabi-splitting phenomenon. We concentrate on the case of the absorption spectrum<sup>16</sup>, which is technically simpler, and also more relevant for the experimental situation of ref.<sup>12</sup>. Consider the case of a system of  $N$  two-level atoms with frequency  $\omega_0$  interacting with a *single*-mode radiation field of frequency  $\omega$ . The relevant quantum-mechanical Hamiltonian is:

$$H = \hbar\omega_0 \sum_i S_i^z + \hbar\omega a^\dagger a + \sum_i (\hbar\lambda S_i^+ a + H.C.) \quad (3)$$

where  $a^\dagger, a$  are the creation and annihilation operators for the cavity radiation-field modes,  $S_i^z, S_i^\pm$  are the usual spin- $\frac{1}{2}$  operators, and  $\lambda$  is the atom-field coupling. The atom-field system is not an isolated system, and there is *dissipation*

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sional 'hits' introduced *ad hoc*. We prefer the motivation for gravity-induced decoherence, which may be mathematically controlled in the context of non-critical strings<sup>7</sup>.

due to the interaction of the system with the surrounding world. One important source of dissipation is the leakage of photons from the cavity at some rate  $\kappa$ . If the rate of dissipation is not too big, a quantum coherent state can still be formed, which would allow the observation of the vacuum-field Rabi oscillations. The density matrix  $\rho$  of the atom-field system obeys a Markov-type master equation for the evolution in time  $t$ <sup>16</sup>:

$$\partial_t \rho = -\frac{i}{\hbar}[H, \rho] - \kappa(a^\dagger a \rho - 2a \rho a^\dagger + \rho a^\dagger a) \quad (4)$$

This is exactly the form of equation proposed<sup>2</sup> as an appropriate description of decoherence effects in quantum gravity.

The limit  $\kappa \ll \lambda\sqrt{N}$  in (4) guarantees the possibility that a coherent quantum state may be formed, i.e., this limit describes environments that are weakly coupled to the system, whose decoherence times (see below) are therefore very long. In this limit, one can concentrate on the off-diagonal elements of the density matrix, and make the following ‘secular’ approximation for their evolution<sup>16</sup>:

$$\partial_t \rho_{ij} = -\frac{i}{\hbar}(E_i - E_j)\rho_{ij} - \Gamma_{ij}\rho_{ij} \quad (5)$$

where  $\Gamma_{ij}$  denotes the damping factor, related to the weak coupling of the atom-field system to the environment. The analysis of ref.<sup>16</sup> pertained to the evaluation of the susceptibility tensor of the system,  $\chi_{\alpha\beta}$ , which can be calculated by considering its interaction with an external field of frequency  $\Omega$ . The absorption spectrum is proportional to  $\text{Im}\chi(\Omega)$ , which has the form

$$\begin{aligned} \text{Im}\chi(\Omega) = & \cos^2\theta \frac{\Gamma_-/\pi}{\Gamma_-^2 + \{\Omega - \omega_0 + \Delta/2 - \frac{1}{2}(\Delta^2 + 4N\lambda^2)^{1/2}\}^2} + \\ & \sin^2\theta \frac{\Gamma_+/\pi}{\Gamma_+^2 + \{\Omega - \omega_0 + \Delta/2 + \frac{1}{2}(\Delta^2 + 4N\lambda^2)^{1/2}\}^2} \end{aligned} \quad (6)$$

with  $\Delta \equiv \omega_0 - \omega$ . In the above expression, the factors  $\Gamma_\pm$  represent the damping in the equation of motion for the off-diagonal element of the density matrix  $\langle \Psi_0 | \rho | \Psi_\pm^{S,C} \rangle$ , where the  $\Psi_\pm$  are eigenfunctions of  $H$ , classified by the eigenvalues of the operators  $S^2$ , and  $S^z + a^\dagger a \equiv C$ . In the  $N$ -atom case under study<sup>16</sup>,  $S = N/2$  and  $C = 1 - N/2$ . The expression (6) summarizes the effect of Rabi vacuum splitting in the absorption spectra of atoms: there is a doublet structure of the absorption spectrum with peaks at:

$$\Omega = \omega_0 - \Delta/2 \pm \frac{1}{2}(\Delta^2 + 4N\lambda^2)^{1/2} \quad (7)$$

For resonant cavities, the splitting occurs with equal weights

$$\Omega = \omega_0 \pm \lambda\sqrt{N} \quad (8)$$

Notice here the *enhancement* in the effect for multiatom systems  $N \gg 1$ . The quantity  $2\lambda$  is called the ‘Rabi frequency’<sup>15</sup>.

There have been simple experiments which have confirmed this effect<sup>14</sup>, involving beams of Rydberg atoms, resonantly coupled to superconducting cavities. The situation which is of interest for the decoherence experiments of ref.<sup>12</sup> involves atoms that are near resonance with the cavity. In this case,  $\Delta \ll \omega_0$  but  $\lambda^2 N / |\Delta|^2 \ll 1$ , so that (7) yields two peaks that are characterized by pure dispersive shifts  $\propto \frac{1}{\Delta}$ :

$$\Omega \simeq \omega_0 \pm \frac{N\lambda^2}{|\Delta|} + \mathcal{O}(\Delta) \quad (9)$$

which is the case in the SC experiment of ref.<sup>12</sup>.

Another important issue, which has been used in ref.<sup>12</sup>, is the dephasing of the atom as a result of the atom-field Rabi entanglement described above. To understand better the situation, we discuss a more generic case, that of a three-state atom,  $f, e, g$ , with energies  $E_g > E_e > E_f$ . Suppose one is interested in the transition  $f \rightarrow e$  by absorption, in the presence of atoms in interaction with a cavity mode. Calling  $D_{ef}^+ \equiv |e \rangle \langle f|$ ,  $D_{ef}^- \equiv |f \rangle \langle e| = (D_{ef}^+)^\dagger$ , we have the following effective Hamiltonian for the transition  $f \rightarrow f$ <sup>17</sup>:

$$H_{eff}^{ef} = \hbar\omega_{eff} D_{ef}^+ D_{ef}^- \quad ; \quad \omega_{eff} = \omega_{ef} + \frac{\lambda^2 n}{\Delta} \quad (10)$$

where the effective dfrequency is due to the dispersive frequency shifts (9) of the Rabi effect, appropriate for near-resonant atom-cavity-field systems. Here  $n$  is the number of cavity photons<sup>d</sup>.

Consider now an experiment to measure, say, the photon number  $n$  in the cavity. The relevant probe  $P$  can be the above-described three-state atom, in a superposition of  $e$  and  $f$  states. In this picture, the photon number  $n$  is an

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<sup>d</sup>The  $\sqrt{n}$  scaling law for the Rabi splitting (8) is also valid in the case of the interaction of a single atom  $n$  cavity oscillator quanta, e.g., in a coherent cavity mode.

eigenvalue of the cavity signal operator  $a_s^\dagger a_s$ , and the interaction Hamiltonian between atom and cavity then reads<sup>17</sup>:

$$H_I = \frac{\hbar\lambda^2}{\Delta} a_s^\dagger a_s D_{ef}^+ D_{ef}^- \quad (11)$$

The probe observable is the atomic dipole operator:

$$A_P = \frac{1}{2i}(D_{ef}^+ - D_{ef}^-) \quad (12)$$

whose Heisenberg evolution equation is

$$i\hbar \frac{d}{dt} A_P = [A_P, H_{eff}^{ef} + H_I] \quad (13)$$

from which it is easily seen that in a time interval  $t$  the phase of the probe changes by an amount:

$$\Delta\phi = \omega_{ef}t + \frac{\lambda^2 n}{\Delta}t \quad (14)$$

The case of interest for the experiment of ref.<sup>12</sup> is a two-state atom. The resulting phase shift is obtained from (14) by setting  $\omega_{ef} = 0$ . Thus, in the experiment of ref.<sup>12</sup>, the phase entanglement due to the atom-field Rabi coupling is

$$\Delta\phi_R = \frac{\lambda^2 n}{\Delta}t \quad (15)$$

for a near-resonance atom-field system, with small detuning  $\Delta$ .

We are now well equipped in to review the experiment of ref.<sup>12</sup> in which a mesoscopic Schrödinger's cat was constructed, and the associated decoherence. The experiment involves sending a Rubidium atom, consisting of two circular Rydberg states  $e$  and  $g$ , through a microwave cavity storing a small coherent field  $|\alpha\rangle$ . The coherent cavity mode is mesoscopic in the sense that an average number of photons is of order  $\mathcal{O}(10)$ . The atom-cavity coupling is measured by the Rabi frequency  $2\lambda/2\pi = 48kHz$ . The condition for Rabi dispersive shifts (9) is satisfied by having  $\Delta/2\pi$  in the range  $[70, 800]kHz$ .

The atom is prepared in the superposition of  $e, g$  states, by means of a resonant microwave cavity  $R_1$ . Then it crosses the cavity  $C$ , which is coupled to a reservoir that dissipates its energy on a characteristic time scale  $T_r \ll 1.5$  ms. A number of photons varying from 0 to 10 is injected by a pulsed source into the cavity  $C$ . The field in the cavity relaxes to vacuum, *dissipating* via

leakage of photons through the cavity, during a time  $T_r$ , before being regenerated for the next atom. The experiment is at an effective temperature of  $T = 0.6K$ , which is low enough that thermal effects are small. After leaving  $C$ , the atom passes through a second cavity  $R_2$ , identical to  $R_1$ . One then measures the probability of finding the atom in the state  $g$ , say. The decoherence time is then measured for various photon numbers. This enables one to test the theoretical predictions that decoherence between two ‘pointer states’ of a quantum superposition occurs at a rate proportional to the square of the distance between the states.

Let us understand this point better. The coherent oscillator states, characterizing the cavity modes, constitute a pointer basis: an oscillator in a coherent state is defined by the average number of oscillator quanta  $n$ :  $|\alpha\rangle$ :  $|\alpha\rangle = \sqrt{n}$ . Then, consider the measurement of the above-described experiment, according to which there is only a phase entanglement between the cavity and the atom. The combined atom-cavity (meter) system is originally in the state

$$|\Psi\rangle = |e, \alpha e^{i\phi}\rangle + |g, \alpha e^{-i\phi}\rangle \quad (16)$$

where the dephasing depends on the atomic level:  $\phi \propto \lambda^2 t / \Delta$ , according to (9), (15). Coupling the oscillator to a reservoir that damps its energy in a characteristic time scale  $T_r$  produces decoherence, which according to the general theory<sup>2,20,17,3</sup> occurs in a time scale inversely proportional to the square of the distance between the ‘pointer’ states  $D^2$ :

$$t_{decoh} = \frac{2T_r}{D^2} \quad (17)$$

In the set up of ref.<sup>12</sup>, the distance  $D$  is given by

$$D = 2\sqrt{n}\sin\phi \simeq 2n^{3/2}\frac{\lambda^2 t}{\Delta} \quad (18)$$

for Rabi couplings  $2\lambda$ , such that  $\lambda^2 t n \ll \Delta$ . For mesoscopic systems,  $n \sim 10$ ,  $D > 1$ , and hence decoherence occurs over a much shorter time scale than  $T_r$ . In particular, for  $\Delta/2\pi \sim 70kHz$ , the decoherence time is  $0.24T_r$ .

This concludes our brief review of the construction of a Schrödinger’s cat, and the associated ‘measurement process’. Notice that the above construction is made in two stages: first it involves an interaction of the atom with the cavity field, which results in a coherent state of the combined ‘atom + meter’, and then *dissipation* is induced by coupling the cavity (measuring apparatus)

to the environment, which damps its energy, thereby inducing *decoherence* in the ‘atom + meter’ system. The important point to realize is that the more macroscopic the cavity mode is, i.e., the higher the number of oscillator quanta, the shorter the decoherence time is. This is exactly what was to be expected from the general theory<sup>4,2,18,7</sup>.

### 3 Quantum Gravity as an Environment, and the Induced Collapse of Wave Functions

We now argue that a similar situation characterizes quantum-gravity vacuum fluctuations. There is a striking analogy between the cavity vacuum and the quantum-gravity one, with its virtual topological fluctuations in space time.

The problem of the interaction of low-energy propagating matter with a dissipative quantum-gravity environment consisting of virtual wormholes<sup>19</sup> was studied in ref.<sup>18</sup>, from a ‘phenomenological’ view point. Coleman had argued that the wormhole state was likely to be a coherent state, and used this argument to support the vanishing of the cosmological constant. However, the coherence assumption was questioned later, and in view of our subsequent studies of the nature of the space-time foam in quantum gravity and/or string theory we expect this not to be the case. However, one can still model the interaction Hamiltonian between operators describing the low-energy probe  $O_P$  and the wormhole state  $|a\rangle$  as<sup>19</sup>

$$H_I \propto O_P(a^\dagger + a) \quad (19)$$

where  $a^\dagger, a$  are creation and annihilation operators for the wormhole state. In the example of ref.<sup>18</sup>,  $O_P$  was taken to be a four-fermion effective interaction

$$O_P \propto \mathcal{O}\left(\frac{1}{m_P^2}\right) \bar{\psi}_1 \gamma^\mu \psi_1 \bar{\psi}_2 \gamma_\mu \psi_2 \quad (20)$$

A low-energy observer has to average out the unobservable wormhole effects, with the result that the low-energy probe  $P$  becomes an *open* system. In ref.<sup>18</sup>, the simple case of a Gaussian distribution for the wormhole configurations was assumed, and the time scale of the induced decoherence of the low-energy probe  $P$  was estimated, using the phenomenological equation for the density matrix suggested in ref.<sup>2</sup>, which was characterized by probability and energy conservation of the probe. In view of our discussion in the previous section, this coupling may be considered as a coupling with only phase damping for the atom. Thus, a sort of Rabi vacuum effect appears, but of course the nature of



the effect is not due to quantum electrodynamics, but due to quantum gravitational interactions. The rôle of the cavity is played by the whole universe, or rather by the microscopic space-time foam<sup>1,2</sup>.

As shown in ref.<sup>18</sup>, the enhancement of the effect for large numbers of atoms, as seen above in our discussion of the simple Rabi vacuum<sup>16</sup> (8) also characterizes the wormhole probe-P coupling. The decoherence of the off-diagonal elements of the density matrix  $\rho(x, x')$ , in a ‘pointer’ basis  $|x\rangle$ , where  $x$  is the center-of-mass location in space time of a system of  $N$  particles is of the form<sup>18</sup>:

$$\rho(X', X, t) \sim \rho_0(X', X, t) \exp[-ND(X' - X)^2 t] \quad (21)$$

where  $D$  represents the coupling of a single particle with a single coherent mode of the wormhole state, and is estimated to be of order  $D \sim m^6/M_P^3$ , in a four-dimensional space-time, for a particle of mass  $m$ , with  $M_P$  the Planck mass. In the estimate (21) a uniform density of wormholes of the order of one per Planck volume in space time was assumed, and all other interactions of the microscopic particles among themselves have been ignored. From (21) one can readily see the characteristic feature that the decoherence rate is proportional to the square of the distance between the pointer states (17), which is a generic feature of Markov-type decoherence<sup>20</sup>.

The wormhole model assumed that on the average energy and probability are conserved. This was also the case in the atom-cavity entanglement case considered above, where there was only a phase entanglement/damping. Such entanglement is capable of producing decoherence by itself, as is clear from the analysis of ref.<sup>18</sup>.

Although the mesoscopic atom + cavity system considered in ref.<sup>12</sup> is also exposed in such quantum-gravity vacuum effects, they are of course much, much weaker than the conventional Rabi coupling, and negligible in the apparatus of ref.<sup>12</sup>. However, as was demonstrated in ref.<sup>18</sup>, in the case of macroscopic systems quantum-gravity effects could conceivably lead to rapid collapse. Unfortunately, it is not possible at present to see such effects in such a cavity experiment. However, experiments to look for macroscopic quantum-gravitational decoherence may become possible in the future, especially in very cold environments, such as SQUIDS<sup>21</sup> or those in which Bose-Einstein condensation has been observed<sup>22</sup>. From our point of view, the experiments of ref.<sup>12</sup>, although due to the conventional quantum field theory of Quantum Electrodynamics in a cavity, are nevertheless fascinating, in that they constitute the first

experimental evidence for the environmentally-induced collapse of a coherent quantum superposition of states.

#### 4 Decoherence in a String Approach to Quantum Gravity

We have argued in ref.<sup>7</sup> that in string quantum gravity there are inherently unobservable *delocalized* modes, carrying information, which fail to decouple from light states in the presence of singular space-time fluctuations. The effective theory of the light states which are measured by *local* scattering experiments can be described by a non-critical Liouville string<sup>23,7</sup>. The zero mode of the Liouville field in such a string theory is identified in ref.<sup>7</sup> with a target time variable. This results in an irreversible temporal evolution in target space, with decoherence and associated entropy production, as we now review.

The effective low-energy theory density matrix is:

$$\tilde{\rho}(local, t) = \int d(delocal) \rho(local, delocal) \quad (22)$$

where  $\tilde{\rho}$  denotes the low-energy density matrix, and the *delocal* states play a role analogous to those of the unseen states  $|B >_I$  inside the black-hole horizon in the arguments of ref.<sup>1</sup>. The integration over *delocal* in (22) ensures that the reduced density matrix  $\tilde{\rho}$  is *mixed* in general, even if the full  $\rho(local, delocal)$  is pure. We have argued that  $\tilde{\rho}$  obeys a modified quantum Liouville equation of the form<sup>7</sup>

$$\partial_t \tilde{\rho} = i[\tilde{\rho}, H] + \delta H \tilde{\rho} \quad : \quad \delta H = -i \Sigma_{i,j} \beta^i G_{ij} [ , g^j] \quad (23)$$

where  $H$  is the usual light-particle Hamiltonian, the indices  $(i, j)$  label all possible microscopically-distinct string background states with coordinate parameters  $g^i$ , and  $G_{ij}$  is a metric in the space of such possible backgrounds<sup>24</sup>. We argue that these are not conformally invariant once one integrates out the *delocal* degrees of freedom, and the  $\beta^i$  are the corresponding renormalization functions. These are non-trivial to the extent that back reaction of the light particles on the background metric cannot be neglected. Equations of the form (23) are quite generic in the context of non-critical string theories<sup>7,25</sup>. We note further that the background fields  $g^i$  must be quantized, as a result of the summation over world-sheet topologies in the Liouville string<sup>7</sup>.

There are general properties of the Liouville system that follow from the renormalizability of the world-sheet  $\sigma$ -model theory<sup>7</sup>. These include *energy*

*conservation* on the average, and *probability* conservation. Specific properties of the renormalization group on the two-dimensional world-sheet<sup>24</sup> entail *monotonic* entropy increase,  $\partial_t S \propto \beta^i G_{ij} \beta^j \geq 0$ , leading to a microscopic arrow of time. The modified quantum Liouville equation (23), can be cast in a form similar to that of the Markov-type evolution (4) of an open quantum-mechanical system:

$$\partial_t \rho = i[\rho, H] - \sum_m \{B_m^\dagger B_m, \rho\}_+ + 2 \sum_m B_m \rho B_m^\dagger \quad (24)$$

where the ‘environment’ operators  $B$  are appropriately-defined ‘square roots’ of the various partitions of the operator  $\beta^i G_{ij} \dots g^j$ <sup>7</sup>.

The maximum magnitude of effect that we can imagine is

$$\delta H \simeq H^2 / M_P \quad (25)$$

which would be around  $10^{-19} \dots 10^{-20}$  GeV for a typical low-energy probe, such as the neutral kaon system. A contribution to the evolution rate equation (23) of this order of magnitude would arise if there were some Planck-scale interaction contributing an amplitude  $A \simeq 1/M_P^2$  and hence a rate  $R \simeq 1/M_P^4$ , to be multiplied by a density  $n \simeq L_P^{-3} \simeq M_P^3$ , yielding the overall factor of  $\simeq 1/M_P$  shown in (25)<sup>18</sup>. Such an estimate was found in a pilot study of a scalar field in a four-dimensional black-hole background<sup>26</sup>, and has also been found in a Liouville-string representation of Dirichlet membranes<sup>27</sup>.

The associated entropy production is a signature of decoherence. Indeed, one can demonstrate in this approach exponential decay in time of the off-diagonal elements of the density matrix in the string theory space  $|g^i\rangle$ . Moreover, the Markov equation (24) implies<sup>28</sup> a stochastic equation of Ito-Langevin type for the state vector  $|\Psi\rangle$  corresponding to the density matrix  $\rho(g^i, t) = \text{Tr}|\Psi\rangle\langle\Psi|$ ,

$$\begin{aligned} |d\Psi\rangle = & -\frac{i}{\hbar} H |\Psi\rangle dt + \sum_m (\langle B_m^\dagger \rangle_\Psi B_m - \frac{1}{2} B_m^\dagger B_m - \\ & \frac{1}{2} \langle B_m^\dagger \rangle_\Psi \langle B_m \rangle_\Psi) |\Psi\rangle dt + \\ & \sum_m (B_m - \langle B_m \rangle_\Psi) |\Psi\rangle d\xi_m \end{aligned} \quad (26)$$

where the  $d\xi_m$  are complex differential random matrices associated with Brownian processes. The advantage of the latter ‘state-vector’ formalism is that it

allows a *localization* of the state vector in an appropriate ‘measurement’ channel, to be identified with a ground state of the string<sup>29</sup>, as a result of the ‘dispersion-entropy minimization’ theorem of ref.<sup>28</sup>.

## 5 Decoherence and CPT Violation

The non-unitary evolution characterising non-critical string theory manifests an arrow of time. Everyday experience tells us that an arrow of time is present macroscopically: our bit (at least) of the Universe is expanding, and we are all of us getting older. On the other hand, no such arrow of time is visible in our accepted fundamental laws of physics: Quantum Field Theory is invariant under CPT, and time  $t$  is just a coordinate in General Relativity - the motion of the Earth around its solar orbit could be reversed with no apparent problem. On the other hand, an arrow of time appears in thermodynamics via the second law, which states that entropy increases monotonically. The arguments of the previous sections raise again the possibility that this could have a microscopic origin.

It has been pointed out in ref.<sup>30</sup> that a microscopic arrow of time must appear if pure states evolve into mixed states as suggested above, in the sense that the strong form of the CPT theorem must be violated. Suppose there is some CPT symmetry transformation  $\Theta$  which maps initial-state density matrices into final-state density matrices:

$$\rho'_{out} = \Theta \rho_{in} \quad (27)$$

and correspondingly

$$\rho'_{in} = \Theta \rho_{out} \quad (28)$$

where

$$\rho_{out} = \mathcal{S} \rho_{in}, \rho'_{out} = \mathcal{S} \rho'_{in} \quad (29)$$

It is easy to deduce from these equations that  $\mathcal{S}$  must have an inverse:

$$\mathcal{S}^{-1} = \Theta^{-1} \mathcal{S} \Theta^{-1} \quad (30)$$

which cannot be true if pure states evolve into mixed states, entropy increases monotonically and the density matrix collapses.

Although there are many people in the quantum gravity community who suspect that some modification of quantum mechanics may be necessary so as to incorporate decoherence associated with black holes, there is disagreement

whether this is necessarily accompanied by CPT violation. This division of opinion is exemplified by the viewpoints of Hawking and Penrose in ref. <sup>31</sup>: Hawking is very reluctant to give up CPT, whereas Penrose accepts it as a likelihood. The formalism we have developed definitely points in the latter direction.

An explicit example where all the above issues are realized has been given in ref. <sup>27</sup>, and will not be repeated here. Even if you do not follow all the arguments leading to the string version of the modified Liouville equation, the latter still provides an interesting phenomenological framework in which one can parametrize possible decoherence and CPT-violating effects with a view to the experimental tests in the neutral kaon system, which are reviewed in the next section.

## 6 Testing Quantum Mechanics and CPT in the Neutral Kaon System

The neutral kaon system has an enviable track record as a probe of fundamental physics, ranging from P violation (the  $\tau$ - $\theta$  puzzle) and CP violation to the motivation for charm coming from the absence of strangeness-changing transitions. It is also known to provide very elegant tests of quantum mechanics, and provides the most stringent available test of CPT at the microscopic level. The formalism of decoherence and related CPT violation developed above can be applied to the neutral kaon system, and experimental upper limits given on such effects.

In our approach, the quantum-mechanical evolution equation is modified to become

$$\partial_t \rho = -i(H\rho - \rho H^\dagger) + \delta H \rho \quad (31)$$

where  $H$  is the conventional quantum-mechanical Hamiltonian, and we can parametrize the modification term  $\delta H$  as <sup>2</sup>

$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \quad (32)$$

where the indices  $\alpha, \beta$  label Pauli matrices  $\sigma_{\alpha,\beta}$  in the  $K_{1,2}$  basis, and we have assumed that  $\delta H$  has  $\Delta S = 0$ . The three free parameters  $\alpha, \beta, \gamma$  must obey the conditions

$$\alpha, \gamma > 0, \quad \alpha\gamma > \beta^2 \quad (33)$$

stemming from the positivity of the matrix  $\rho$ .

It is easy to see that these parameters induce decoherence and violate CPT<sup>8</sup>. Various observables sensitive to these parameters have been discussed in the literature, including the following asymmetries, which have already been used in experimental probes of this formalism: the  $2\pi$  decay asymmetry

$$A_{2\pi} \equiv \frac{\text{Tr}(O_{2\pi}\bar{\rho}(t)) - \text{Tr}(O_{2\pi}\rho(t))}{\text{Tr}(O_{2\pi}\bar{\rho}(t)) + \text{Tr}(O_{2\pi}\rho(t))} \quad (34)$$

where  $O_{2\pi}$  is an observable measuring the rate of  $2\pi$  decay, and  $\rho, \bar{\rho}$  denote the density matrices of states that are tagged initially as pure  $K, \bar{K}$  respectively, and the double semileptonic decay asymmetry

$$A_{\Delta m} = \frac{R(K^0 \rightarrow \pi^+) + R(\bar{K}^0 \rightarrow \pi^-) - R(\bar{K}^0 \rightarrow \pi^+) - R(K^0 \rightarrow \pi^-)}{R(K^0 \rightarrow \pi^+) + R(\bar{K}^0 \rightarrow \pi^-) + R(\bar{K}^0 \rightarrow \pi^+) + R(K^0 \rightarrow \pi^-)} \quad (35)$$

in which various systematic effects cancel. The asymmetry  $A_{2\pi}$  is sensitive to the presence of  $\alpha, \beta$  and  $\gamma$ , whereas  $A_{\Delta m}$  is particularly sensitive to  $\alpha$ . These and other measurements would enable the form of decohering CPT violation that we propose here to be distinguished in principle from “conventional” quantum-mechanical CPT violation<sup>9,32</sup>.

Together with the CPLEAR collaboration itself, we have published a joint analysis of CPLEAR data<sup>10</sup>, constraining the CPT-violating parameters  $\alpha, \beta, \gamma$ . The data for  $A_{2\pi}$  and  $A_{\Delta m}$  agree perfectly with a conventional quantum-mechanical fit, and provide the following upper limits when we impose the positivity constraints (33):

$$\alpha < 4.0 \times 10^{-17} \text{GeV}, \quad \beta < 2.3 \times 10^{-19} \text{GeV}, \quad \gamma < 3.7 \times 10^{-21} \text{GeV} \quad (36)$$

We cannot help being impressed that these bounds are in the ballpark of  $m_K^2/M_P$ , which is the maximum magnitude that we could expect any such effect to have.

## 7 Outlook

The experimental verification of environmentally-induced decoherence, observed in the mesoscopic atom + cavity systems of ref.<sup>12</sup>, has opened the way for an understanding of the transition from the quantum to classical worlds, as anticipated by theorists for a long time<sup>4,2,18,7</sup>. This experiment, although based on conventional QED environmentally-induced decoherence, is important for

its analogies with the decoherence that may result from quantum-gravity vacuum fluctuations. Simple laws of scaling with the number of microscopic constituents suggest that couplings between the quantum-gravity vacuum and low-energy probes might allow an observable enhancement of the gravitational decohering effects in macroscopic systems. If true, this could open the way for an understanding of the nature of quantum space time.

We have discussed the density-matrix formalism of open systems, and applied it specifically to the neutral kaon system, which is believed to be the most sensitive probe of quantum mechanics to date. Our approach and formalism can in principle be distinguished from others by measuring a number of different  $K, \bar{K}$  decay asymmetries<sup>9</sup>. We can offer our experimental colleagues no guarantee of success in such an experimental programme. Nevertheless, we think that the importance of the issues discussed here motivate a new series of microscopic experiments<sup>33</sup> to test quantum mechanics and CPT.

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